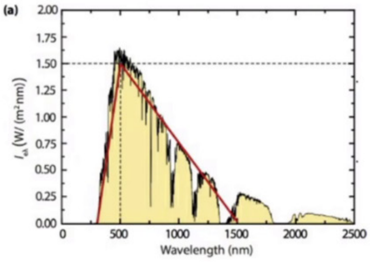
Unofficial Solutions of “Solar Energy” Tutorial 4 (WT2021/22)

These are screenshots of the sample solution (that is not handed out to the students) taken from a hybrid tutorial.

Exercise 1



Spectral irradiance is the **irradiance of a surface <u>per unit frequency or wavelength</u>**, depending on whether the spectrum is taken as a function of frequency or of wavelength

The spectral irradiance is given as a function of wavelength λ, and **gives the power (energy per unit time) received by the surface for a particular wavelength of light**.

- 1a. Calculate the total irradiance of the solar simulator

(a) Answer: $I_e = 900\,\mathrm{W\,m^{-2}}$

The irradiance of the solar simulator is calculated by integrating the spectral irradiance over wavelength:

$$I_e = \int_0^\lambda I_{e\lambda}\,d\lambda = \int_{300\,\mathrm{nm}}^{500\,\mathrm{nm}} \left(7.5\cdot 10^{15}\lambda - 2.25\cdot 10^9\right) d\lambda + \int_{500\,\mathrm{nm}}^{1500\,\mathrm{nm}} \left(-1.5\cdot 10^{15}\lambda - 2.25\cdot 10^9\right) d\lambda$$

$$=7.5\cdot 10^{15}\times\frac{(500\cdot 10^{-9})^2-(300\cdot 10^{-9})^2}{2} - 2.25\cdot 10^9\times(500\cdot 10^{-9} - 300\cdot 10^{-9}) +$$

$$2.25\cdot 10^9\times(1500\cdot 10^{-9} - 500\cdot 10^{-9}) - 1.5\cdot 10^{15}\times\frac{(1500\cdot 10^{-9})^2-(500\cdot 10^{-9})^2}{2}$$

$$= 900\,\mathrm{W\,m^{-2}}$$

Alternatively (and much easier), we can just calculate the area under the red triangle:

$$I_e = \frac{200\cdot 10^{-9}\times 1.5\cdot 10^9}{2} + \frac{1000\cdot 10^{-9}\times 1.5\cdot 10^9}{2} = 900\,\mathrm{W\,m^{-2}}$$

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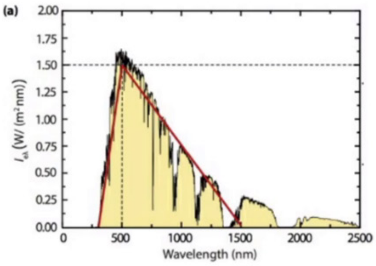

The quantum flux (also called photon flux) is defined as the **number of photons (in µmol) per second and unit area on a surface**

$E = h\nu = \frac{hc}{\lambda}$ (γ) I ρ A

$\phi = \frac{\text{Intensity}}{\text{Energy of a Photon}} = \frac{I}{E}$

$\phi = \frac{I\lambda}{hc}$ —— (1)

- 1b. What is the photon flux of the solar simulator?

(b) Answer: $\Phi_{ph} = 3.48\cdot 10^{21}\,\mathrm{s^{-1}\,m^{-2}}$

Applying the given formula for photon flux:

$$\Phi_{ph} = \int_0^\lambda \Phi_{ph\lambda}\,d\lambda = \int_0^\lambda I_{e\lambda}\frac{\lambda}{hc}\,d\lambda$$

The wavelength was multiplied with the spectral irradiance before integration

$$= \frac{1}{hc}\left[\int_{300\,\mathrm{nm}}^{500\,\mathrm{nm}}(7.5\cdot 10^{15}\lambda^2 - 2.25\cdot 10^9\lambda)\,d\lambda + \int_{500\,\mathrm{nm}}^{1500\,\mathrm{nm}}(2.25\cdot 10^9\lambda - 1.5\cdot 10^{15}\lambda^2)\,d\lambda\right]$$

$$= \frac{1}{hc}\left[\left[7.5\cdot 10^{15}\frac{\lambda^3}{3} - 2.25\cdot 10^9\frac{\lambda^2}{2}\right]_{300\,\mathrm{nm}}^{500\,\mathrm{nm}} + \left[2.25\cdot 10^9\frac{\lambda^2}{2} - 1.5\cdot 10^{15}\frac{\lambda^3}{3}\right]_{500\,\mathrm{nm}}^{1500\,\mathrm{nm}}\right]$$

$$= 0.33\cdot 10^{21}\,\mathrm{s^{-1}\,m^{-2}} + 3.15\cdot 10^{21}\,\mathrm{s^{-1}\,m^{-2}} = 3.48\cdot 10^{21}\,\mathrm{s^{-1}\,m^{-2}}$$

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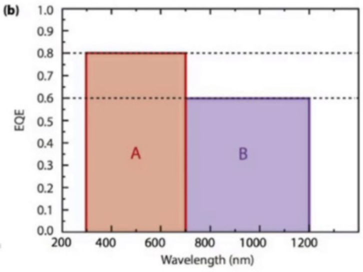
1c. Which junction acts like a top cell in the tandem cell A or B?

Answer: Junction A



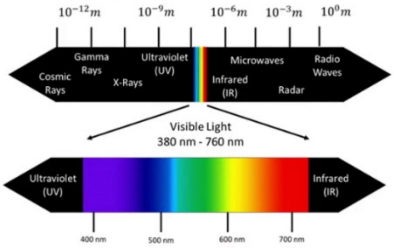
$I = \phi \cdot E_{photon}$

$E_{photon} = hc/\lambda$

More energetic photons have lower wavelength (near UV ray) = smaller penetration depth in materials

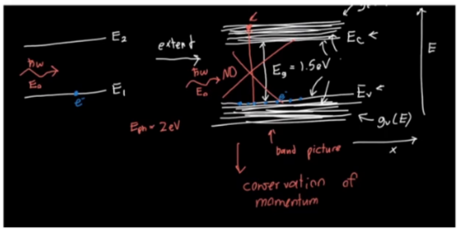
Tandem or multi-junction solar cells utilize two or more photovoltaic absorbers each with different properties. The combined solar cells with different bandgaps into "tandem stack", allows the sun's spectrum can be more utilized.

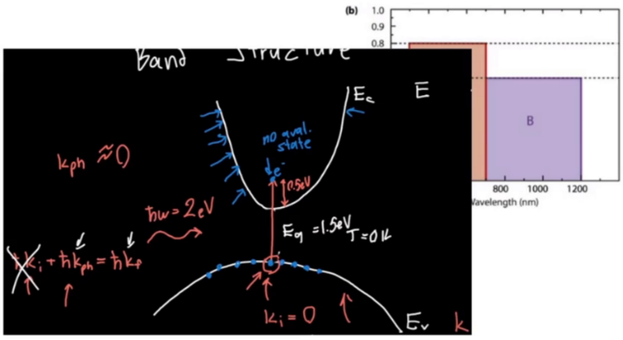
<u>As a rule, the solar cells at the top of the monolithic stack have a large bandgap</u> and convert UV and blue light into electricity, while the solar cells at the bottom of the stack have smaller bandgaps (eVs) and convert the red and IR light efficiently into electricity



Junction A has the highest band gap of the two junctions. Since the more energetic photons have smaller penetration depth in materials, the high band gap junction should always act as the top cell. Similarly, less energetic photons (near infrared light) have larger penetration depths. Therefore the bottom cell should be the cell with the lowest band gap. The bottom cell will then collect the photons that have not been collected by the top cell since their energy was not larger than the band gap of cell A.

1d. What is the bandgap of the absorber layer of junction A?





2 level system with 2 energy levels:

Eph = hf

Semiconductor with band of energy levels

Each energy levels in each band also has a specific momentum, And the momentum needs to be conserved while the electrons transition take place

Band structure: to represent energy of different states: its an Energy – wavenumber plot.

The band gap energy (Eg)= the minimum energy needed to excite an electron from the VB to the CB

A semiconductor will <u>not absorb photons of energy less than the band gap</u>

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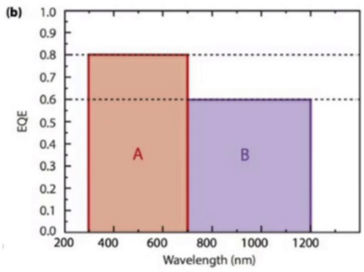
1d. What is the bandgap of the absorber layer of junction A?



(d) Answer: $E_G = 1.77\,\text{eV}$

The energy of a photon with wavelength λ in eV is given by:

$$E_{\text{ph}} = \frac{hc}{\lambda q}$$

Using this formula for wavelength $\lambda = 700\,\text{nm}$ will give:

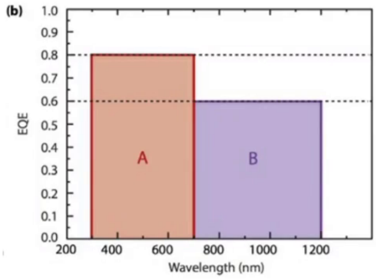
$$E_G = \frac{6.626 \cdot 10^{-34}\,\text{J s} \times 2.998 \cdot 10^{8}\,\text{m s}^{-1}}{700\,\text{nm} \times 1.602 \cdot 10^{-19}\,\text{C}} = 1.77\,\text{eV}$$

Note that you have to use the highest wavelength of junction A since this will give the lowest energy needed to excite electrons (the band gap energy).

The energy of the electron-hole pair produced by a photon is equal to the bandgap energy.

1eV = q = 1.602×10^{-19} J

1e. Calculate the short circuit current density J_{sc} of junction A if the solar cell is measured under the spectrum provided by the solar simulator



•**External Quantum Efficiency (EQE)=** ratio of the number of charge carriers collected by the solar cell to the number of photons of a given energy *shining on the solar cell from outside* (incident photons).

EQE =

$$\frac{\text{current flowing through the cell } (I = q/t) = \text{electrons per sec}}{\text{incident Photons per sec}}$$

The short-circuit current density J_{sc} that can be obtained from the standard 1000 W/m² solar spectrum (AM1.5)

•**AM1.5**
Solar panels do not generally operate under exactly one atmosphere's thickness: if the sun is at an angle to the Earth's surface the effective thickness will be greater.

Many of the world's major population centres, and hence solar installations and industry, across Europe, China, Japan, the United States of America and elsewhere (including northern India, southern Africa and Australia) lie in temperate (moderate) latitudes. An AM number representing the spectrum at mid-latitudes is therefore much more common.

"AM1.5", 1.5 atmosphere thickness, corresponds to a solar zenith angle of = 48.2°.
The short-circuit current density J_{se} that can be obtained from the standard 1000 W/m² solar spectrum (AM1.5).

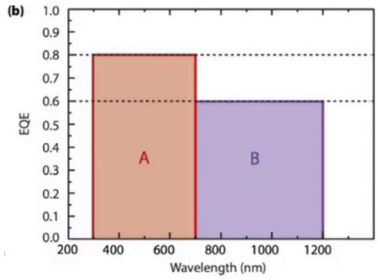
AM1.5 = Air Mass 1.5 spectrum: refers to exactly two standard terrestrial solar spectral irradiance spectra**: standard direct normal + standard global(or total) spectral irradiance**

The illuminance for Daylight under A.M.1.5 is given as **109,870 lux** (corresponding with the A.M. 1.5 spectrum to **1000.4 W/m²**).

The two spectra define a standard direct normal spectral irradiance and a standard total (global, hemispherical, within steradian field of view of the tilted plane at. from horizontal) spectral irradiance 20

1e. Calculate the short circuit current density J_{sc} of junction A if the solar cell is measured under the spectrum provided by the solar simulator

$$j_{sc} = \int_{\lambda 1}^{\lambda 2} q\phi_{ph}(\lambda).EQE(\lambda).d\lambda$$



The short-circuit current is the current through the solar cell when the voltage across the solar cell is zero.

The short circuit current density jsc = short circuit current per area A of the solar cell).

derived from the charge carrier generation rate and the collection probability for generated charge carriers by integrating over the thickness of the solar cell

(e) Answer: $J_{sc} = 14.6\,\text{mA cm}^{-2}$

The short circuit current density can be calculated as:

$$J_{sc} = q\int \text{EQE}(\lambda)\Phi_{ph,\lambda}^{AM1.5}\,d\lambda = q\int \text{EQE}(\lambda) I_{e\lambda}\frac{\lambda}{hc}\,d\lambda$$

The EQE of junction A is 0.8 for $300\,\text{nm} < \lambda < 700\,\text{nm}$, which can be seen from the graph. This gives:

$$J_{sc} = \frac{q}{hc} \times 0.8 \int_{300\,\text{nm}}^{700\,\text{nm}} I_{e\lambda}\lambda\,d\lambda$$

$$= \frac{0.8q}{hc}\left[\int_{300\,\text{nm}}^{500\,\text{nm}} (7.5\cdot 10^{15}\lambda^2 - 2.25\cdot 10^9\lambda)\,d\lambda + \int_{500\,\text{nm}}^{700\,\text{nm}} (2.25\cdot 10^9\lambda - 1.5\cdot 10^{15}\lambda^2)\,d\lambda\right]$$

$$= \frac{0.8q}{hc}\left[\left[7.5\cdot 10^{15}\frac{\lambda^3}{3} - 2.25\cdot 10^9\frac{\lambda^2}{2}\right]_{300\,\text{nm}}^{500\,\text{nm}} + \left[2.25\cdot 10^9\frac{\lambda^2}{2} - 1.5\cdot 10^{15}\frac{\lambda^3}{3}\right]_{500\,\text{nm}}^{700\,\text{nm}}\right]$$

$$= 41.9\,\text{A m}^{-2} + 103.9\,\text{A m}^{-2} = 14.6\,\text{mA cm}^{-2}$$

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Exercise 2

(a) Answer: $V_{oc} = 32.4\,\text{V}$

If all the cells are in series, the voltage will be added to get the open circuit voltage of the module.

$$V_{oc} = 0.6\,\text{V} \times 54 = 32.4\,\text{V}$$

(b) Answer: $I_{sc} = 4\,\text{A}$

The short circuit current of a solar module with all the solar cells connected in series is the same as the solar cell with the lowest current. At STC, that means that the short circuit current will be 4 A.

(c) Answer: $I_{sc} = 2\,\text{A}$

If there is partial shading, the shaded cell will lower the current of the whole module to 2 A. That is why bypass diodes are introduced, to include an alternative path for the current of non-shaded cells when some of them are shaded.

Exercise 3

(a) Shading of individual cells results in a lower short circuit current I_{sc} since the illuminated area of the cell is smaller. The open circuit voltage V_{oc} under shading, with the assumption that $I_{sc} = I_{ph}$, can be calculated as:

$$V_{oc} = \frac{k_B T}{q}\ln\left(\frac{I_{sc}}{I_0} + 1\right)$$

$$I_{sc} = I_0 . e^{\left(\frac{qV_{oc}}{k_B T} - 1\right)}$$

This equation is also used to determine the saturation current I_0 of all cells:

-1 outside exp

$$I_0 = \frac{I_{sc}}{\exp\left(\frac{qV_{oc}}{k_B T} - 1\right)} \approx \frac{6\,\text{A}}{\exp\left(\frac{1.602\cdot 10^{-19}\,\text{C} \times 0.68\,\text{V}}{1.38\cdot 10^{-23}\,\text{m}^2\,\text{kg}\,\text{s}^{-2}\,\text{K}^{-1} \times 300\,\text{K}}\right)} = 1.937\cdot 10^{-11}\,\text{A}$$

The I-V curves of the module are calculated with the ideal diode equation:

$$I(V) = I_0\left[\exp\left(\frac{qV}{k_B T}\right) - 1\right] - I_{sc}$$

There are 10 cells which are not shaded, which have $V_{oc} = 0.68\,V$ and $I_{sc} = 6\,A$. The module *I-V* curve is obtained by summing the current of the individual shaded and unshaded solar cell curves, taking into account the voltage lost in the bypass diodes for the shaded solar cells.

Isc = 6A
Voc of 1 cell (unshaded) = 0.68 V
Voc of 10 cell (unshaded) = **6.8 V**

There are 20 cells which are shaded 30% of their area, this gives:

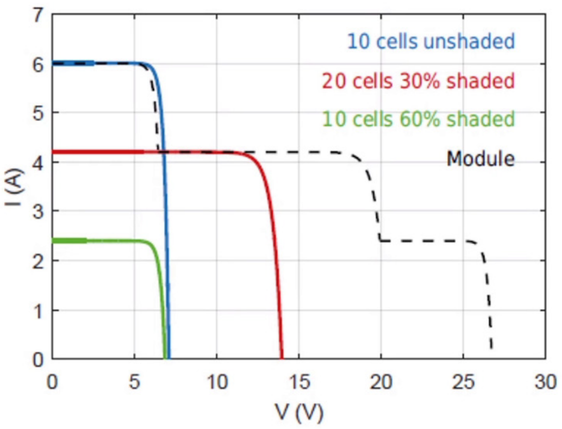
$$I_{sc} = 6\,A \times (1 - 0.3) = 4.2\,A$$
$$V_{oc} = \frac{1.38 \cdot 10^{-23}\,m^2\,kg\,s^{-2}\,K^{-1} \times 300\,K}{1.602 \cdot 10^{-19}\,C} \times \ln\left(\frac{4.2\,A}{1.937 \cdot 10^{-11}\,A} + 1\right) = 0.671\,V$$

Isc = 4.2 A
Voc of 1 cell (30% shaded) = 0.671 V
Voc of 20 cell (30% shaded) = **13.42 V**
k= Boltzmann's constant

There are 10 cells which are shaded 60% of their area, this gives:

$$I_{sc} = 6\,A \times (1 - 0.6) = 2.4\,A$$
$$V_{oc} = \frac{1.38 \cdot 10^{-23}\,m^2\,kg\,s^{-2}\,K^{-1} \times 300\,K}{1.602 \cdot 10^{-19}\,C} \times \ln\left(\frac{2.4\,A}{1.937 \cdot 10^{-11}\,A} + 1\right) = 0.656\,V$$

Isc = 2.4 A
Voc of 1 cell (60% shaded) = 0.656 V
Voc of 10 cell (60% shaded) = **6.56 V**



I-V curve of the module: Joining all the three curves together in series.

Figure 15.1: Module IV curve and shaded/unshaded solar cells.

(b) Answer: $P = 0.72\,W$

The total power lost in the bypass diodes is calculated by multiplying the constant forward voltage of 10 mV of each bypass diode with the current lost by shading.

20 cells, 30% shaded: $I = 6\,A - 4.2\,A = 1.8\,A$, $V = 10\,mV \times 20 = 0.2\,V$
10 cells, 60% shaded: $I = 6\,A - 2.4\,A = 3.6\,A$, $V = 10\,mV \times 10 = 0.1\,V$
$P = 1.8\,A \times 0.2\,V + 3.6\,A \times 0.1\,V = 0.72\,W$

(c) Answer: 18 solar cells

In the worst case, when n solar cells are connected in series and only 1 cell is shaded, the voltage of the shaded cell V_s should be lower than the breakdown voltage $V_{breakdown}$. This voltage is calculated as:

$$V_s = V_F + (n-1) \times V_{oc} = 0.01\,V + (n-1) \times 0.68\,V$$

With V_F the constant forward voltage of the bypass diode.

$$V_s = n \times 0.68\,V - 0.67\,V < 12\,V, \quad n = \frac{12\,V + 0.67\,V}{0.68\,V} = 18.6$$

Thus every 18 cells should have a bypass diode.

Exercise 4

Answer: 6 steps

The MPPT should reach point M at 22 V starting from point A at 8 V, considering its two modes of operation.

1. In the first step, the MPPT moves the operating point by a coarse adjustment to point B, corresponding to a voltage of $8\,\mathrm{V} + 5\,\mathrm{V} = 13\,\mathrm{V}$.
2. As the power increased in going from A to B, the next step is a coarse one. So the MPPT moves the operating point to C, corresponding to a voltage of $13\,\mathrm{V} + 5\,\mathrm{V} = 18\,\mathrm{V}$.
3. As the power increased in going from B to C, the next step is a coarse one. So the MPPT moves the operating point to D, corresponding to a voltage of $18\,\mathrm{V} + 5\,\mathrm{V} = 23\,\mathrm{V}$.
4. The operating point D has now crossed the MPP, but the tracker does not know this yet. As the power increased in going from C to D, the next step is a coarse one. So the MPPT moves the operating point to E, corresponding to a voltage of $23\,\mathrm{V} + 5\,\mathrm{V} = 28\,\mathrm{V}$.
5. As the move from D to E has drastically reduced the PV power, the MPPT reverses direction and performs a fine adjustment. Thus, it moves the operating point to F, corresponding to a voltage of $28\,\mathrm{V} - 1\,\mathrm{V} = 27\,\mathrm{V}$.
6. As the move from E to F has increased the power, the next step is a coarse one. So, the operating point moves to M, corresponding to a voltage of $27\,\mathrm{V} - 5\,\mathrm{V} = 22\,\mathrm{V}$.

This is V_{mpp}, the MPPT has successfully tracked the MPP in 6 steps.

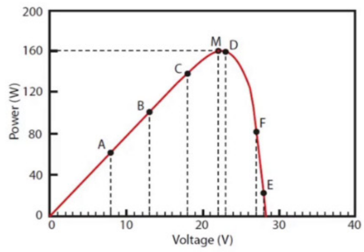


Figure 19.1: Perturb and Observe MPPT with two modes of operation.

Exercise 5

The battery C Rating is **the measurement of current in which a battery is charged and discharged at**.

1C Rate (1C current), = rated capacity of the battery
= the current (Amps) that can discharge a fully charged in 1 hour.

2C Rate (2C current), = the current (Amps) that can discharge a fully charged in ½ hour.

(a) Answer: $E_{\mathrm{bat}} = 1200\,\mathrm{Wh}$

The energy capacity of a battery is given by multiplying the rated battery voltage by the battery capacity C_{bat}:

$$E_{\mathrm{bat}} = C_{\mathrm{bat}} V = 100\,\mathrm{Ah} \times 12\,\mathrm{V} = 1200\,\mathrm{Wh}$$

(b) Answer: $I = 200\,\mathrm{A}$

A C-rate of 1C (dis)charges the entire battery in 1 hour, and therefore corresponds to a current of $I = 100\,\mathrm{A}$. Consequently, a C-rate of 2C (dis)charges the entire battery in 0.5 hour, and therefore corresponds to a current of $I = 200\,\mathrm{A}$.

(c) Answer: 18 minutes

A C-rate of 2C (dis)charges the entire battery in 0.5 hour. So 60% of the battery is charged in $60\% \times 0.5\,\mathrm{h} = 18$ minutes. 200 A = 30 mins (fully charged from 0-100%)
60% charge = 60% (30 mins). Assuming linearly charged

(d) Answer: $\eta_{\mathrm{bat}} = 81\%$

The battery efficiency is defined as the product of voltaic and coulombic efficiency:

$$\eta_{\mathrm{bat}} = \eta_V \times \eta_C = 90\% \times 90\% = 81\%$$